The Association
of
Engineering and Shipbuilding Draughtsmen

DYNAMIC BALANCING

By A. BELL, B.Sc. (Hons.)

Published by The Draughtsman Publishing Co., Ltd.,
"Mansefield," Market Street, Rugeley, Staffordshire.

SESSION 1943-44.

Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.

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INTRODUCTORY.

In the design of machinery, it should be ensured, as far as possible, that no unnecessary vibration is present when the machinery is operating, as vibrations may contribute to the fracture of certain parts of the mechanism or structure. This is particularly important in the case of high-speed machinery, such as steam turbines, high-speed internal combustion engines, etc.

Vibrations may arise from any of a number of sources, common examples being unbalance of rotating parts about their axis of rotation, torsional oscillations of shafts, and inertia forces inherent in reciprocating mechanisms.

It is the purpose of this paper to show how the first of these examples, the unbalance of rotating parts, may be dealt with in the design of the machinery, and also to show what means exist to rectify unbalance due to inaccuracies of manufacture.

Centrifugal and Centripetal Force.

Considering first a mass of weight W lbs., moving in a circular path with constant angular velocity w radians per second (see Fig. 1), there is a centrifugal force acting at all times outwards from the centre of rotation and through the centre of gravity of the rotating mass. The magnitude of this force F is given by

$$F = \frac{W}{g} w^2 r \text{ lbs.}$$

where r is the radius, in feet, of the circle described by the centre of gravity of the mass.

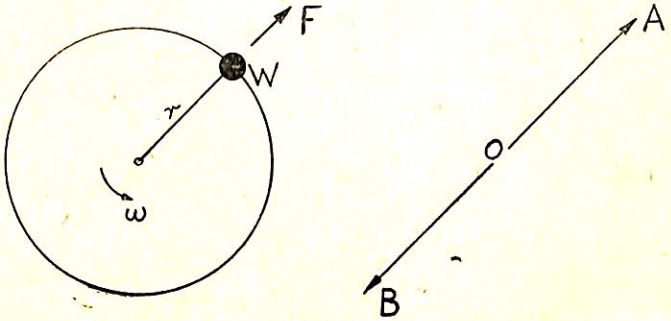

Fig. 1.

There must be a force equal to, and acting in direct opposition to this centrifugal force, constraining the mass to its circular path. This force is known as the centripetal force, and is the force exerted by the structure or supports of the system on the rotating mass. Examples of the way in which this centripetal force can exist are the reactions between the flanges of the wheels of a railway locomotive and the rails when travelling round a curve ; and the tension in a cord when a weight is being whirled at its end.

Now, forces may be represented by vectors, since they have both magnitude and direction, and in Fig. 1 the vectors representing the centrifugal and centripetal forces are shown by OA and OB respectively, the lengths of OA and OB representing the magnitude of the force F, the directions being parallel to the line through the centre of rotation and the centre of gravity of the mass.

In the case of a rotating shaft carrying a mass (such as a pulley), whose centre of gravity does not lie on the axis of rotation of the shaft, the centrifugal force acts radially outwards from the centre of rotation through the centre of gravity of the pulley, and as the shaft rotates, so does the direction of this force. The centripetal force exists as the reactions between the shaft and its bearings, the centripetal force also rotating with the shaft.

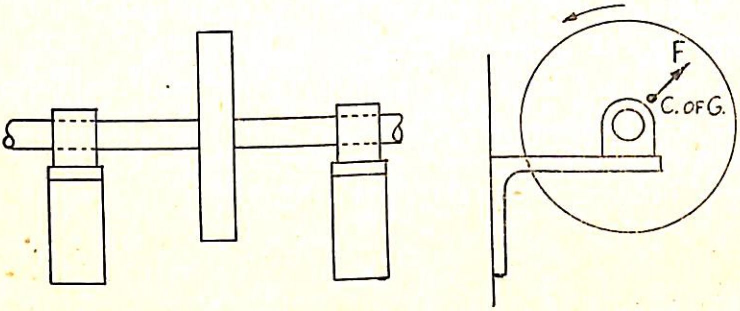

Fig. 2.

In Fig. 2, such a case is illustrated, the bearings being mounted on a cantilever structure. Now the centrifugal force will be acting upwards at the start of a revolution and downwards a half revolution later, completing the cycle at the end of a complete revolution of the shaft. Therefore, the supports, in addition to carrying the normal working load, will be subjected to an oscillating load, the frequency of which will be equal to the speed of the shaft. Should this frequency approach the natural frequency of vibration of the support, then the vibrations of the support may reach dangerous proportions, possibly resulting in fracture of the support.

Another source of danger occurs when a shaft carrying a mass whose centre of gravity is not on the axis of rotation is run up to high speeds. The frequency of the changes in direction of the centrifugal force may approach the natural frequency of the shaft oscillating transversely between its bearings, in which case the amplitude of this oscillation can reach dangerous proportions. The speed at which this takes place is known as the "Whirling Speed."

Apart from these considerations, it can be seen that the loads on the bearings of an unbalanced shaft will not be constant, but will vary rapidly in both magnitude and direction, which in itself is a bad feature.

It can be realised that in all cases it is advisable to make sure that a system of rotating masses does not produce vibratory loads on its supports, and in some cases it is indeed essential that they should not. The following shows how systems of masses, rotating about a common axis, may be balanced by the introduction of balancing masses to the system.

Balancing of a Single Mass by a Second Mass in the same Plane of Rotation.

Referring to Fig. 3, a mass of weight W_1 lbs. is attached to a disc rotating about its centre O. The radius of rotation of the centre of gravity of W_1 is r_1 feet, and the angular velocity of the disc is w radians per second.

Then the centrifugal force F_1 exerted on the disc by W_1 is

$$\frac{W_1}{g} w^2 r_1 \text{ lbs.},$$

acting from O through the centre of gravity of W_1.

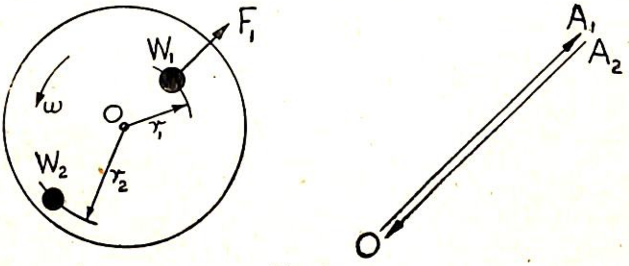

Fig. 3.

To balance this force, it is necessary to add a mass to the disc, so that the centrifugal force exerted by this second mass is equal in magnitude but opposite in direction to F_1, thus making the resultant force on the disc zero.

Let this mass be W_2 lbs. in weight, and r_2 the distance of its centre of gravity from the axis of rotation, W_2 being diametrically opposite to W_1.

$$\text{Then centrifugal force} \quad F_2 = \frac{W_2}{g} w^2 r_2 \text{ lbs.}$$

and since $F_1 = F_2$

$$\frac{W_1}{g} w^2 r_1 = \frac{W_2}{g} w^2 r_2$$

$$\text{or} \quad W_1 r_1 = W_2 r_2$$

It is seen, therefore, that for balance, the products of the weights and the distances of their centres of gravity from the axis must be equal, since these products are proportional to the centrifugal forces.

For example, if a weight of 10 lbs. is rotating at a radius of 1 foot, it can be balanced by a weight of 20 lbs. at 6 inches radius, or 5 lbs. at 2 feet radius, or any weight at a radius at which the product of its weight and radius equals 10 lbs. ft.

A vector diagram of the centrifugal forces is included in Fig. 3.

The length of OA_1 represents the product $W_1 r_1$ and is plotted in the direction given by a line from the centre of the disc through the centre of gravity of W_1.

A_2O represents the product $W_2 r_2$ in the opposite direction. (Actually, OA_1 and A_2O would lie on top of each other, since they both act through the centre of the disc, but they are shown side by side for convenience).

Since the vector diagram closes, the two forces F_1 and F_2 are in equilibrium.

Definitions.

In what follows, the terms used as under are to have the meanings ascribed hereto :—

"Radius of rotation" is to be taken as meaning the radius of the circle which the centre of gravity of the rotating mass describes about the axis of rotation.

"Plane of rotation" is to be taken as the plane in which the centre of gravity of the rotating mass moves.

"Centre of rotation" is to be taken as the intersection of the axis of rotation with the plane of rotation.

"Direction of the centrifugal force" in all cases is the direction of a line from the centre of rotation through the centre of gravity of the rotating mass.

All weights and distances are to be taken as being in pounds and feet respectively, angular velocities being in radians per second.

It might be conveniently stated here, that the method of approach to these balancing problems is in every case to assume that the shaft has no rotary motion, for the purposes of laying out the vector diagrams, but that the centrifugal forces which arise, due to rotary motion, are acting on the shaft in their respective directions.

Balancing of Two Masses in the same Plane of Rotation by a Third Mass in the same Plane.

Referring to Fig. 4, a perfectly-balanced disc rotating about its centre O, carries two masses of weight W_1 and W_2 respectively, the radii of rotation of the masses being r_1 and r_2.

W_3 is the mass (at radius r_3) which is intended to balance W_1 and W_2. If the angular velocity of the disc is w, the forces acting are :

$$F_1 = \frac{W_1}{g} w^2 r_1 ; \qquad F_2 = \frac{W_2}{g} w^2 r_2 ;$$

$$\text{and } F_3 = \frac{W_3}{g} w^2 r_3 \text{ lbs.}$$

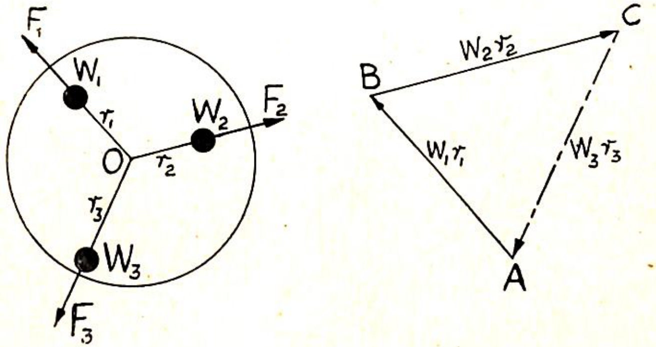

Fig. 4.

These may be represented by the products W_1r_1, W_2r_2, and W_3r_3 respectively, since the term w^2/g is common to each.

The most convenient method of solving the problem is by using a vector diagram as shown.

Set out AB in the direction of the centrifugal force F_1 so that the length of AB represents the product $W_1 r_1$ to some convenient scale. Then in the direction of the force F_2, set out BC such that BC represents $W_2 r_2$ to the same scale.

Then the line AC is the vector representing the resultant of F_1 and F_2, and the centrifugal force F_3 exerted by the mass W_3 must be equal and opposite to AC, so that the resultant of F_1, F_2 and F_3 shall be zero. Therefore CA is the vector representing the product $W_3 r_3$.

If the radius at which the balancing mass is to be attached is fixed, then the required weight W_3 is obtained by dividing the quantity represented by CA by the radius. Conversely, for a fixed mass, the radius is obtained by dividing the quantity CA by the weight of the mass. The angular position at which the mass should be attached is given by the direction of CA, *i.e.*, the mass W_3 should be attached in a position such that the direction of the centrifugal force F_3 is parallel to CA.

Since the vector diagram closes, the system must be in balance.

Balancing of any number of Masses in the same Plane of Rotation by another Mass in the same Plane.

The foregoing principles may be applied to any system of rigidly connected masses, providing that their centres of gravity all rotate in the same plane.

In Fig. 5, masses of weight W_1, W_2, W_3 and W_4 are rotating at radii of r_1, r_2, r_3 and r_4 respectively, their angular positions relative to each other being fixed. To balance this system, proceed as follows.

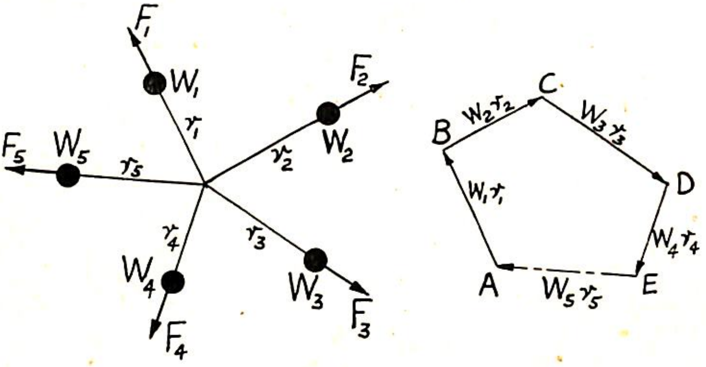

Fig. 5.

Set out AB parallel to centrifugal force F_1 such that the length of AB represents the product $W_1 r_1$ to some convenient scale. Then continue with BC parallel to the centrifugal force F_2, such that the length of BC represents $W_2 r_2$ to the same scale. Similarly, $W_3 r_3$ and $W_4 r_4$ are represented by the vectors CD and DE.

The vector AE then represents the resultant out of balance force of the system, and the force necessary to balance the system may be applied by placing a mass of weight W_5 at a radius of r_5, such that the product $W_5 r_5$ equals the quantity represented by EA, and also such that the direction of the centrifugal force F_5 is parallel to the direction EA.

It should be realised that the addition of a balancing mass to a system of rotating masses does not eliminate the stresses caused by the centrifugal forces. For example, if a mass of 10 lbs. is rotating with angular velocity 10 radians per second at a radius of 2 feet, on a bar pivoted at the mid-point of its length, the load on the pivot will be 62 lbs. Now, by adding a similar mass at the same radius, but at the opposite side of the pivot, the resultant load on the pivot will be zero, but the tension in the bar will still be 62 lbs., the function of the balancing mass being merely to provide a force equal to the load carried by the pivot.

It may be noticed that the method of balancing consists of nothing more than adding a mass to the system such that the centre of gravity of the whole system is brought on to the axis of rotation. However, in cases where the unbalanced masses are not in the same plane of rotation, this is not sufficient, as is now to be explained.

Statically-Balanced Masses not in Dynamic Balance.

In the cases dealt with so far, the centres of gravity of the individual masses have all been in the same plane of rotation, but Fig. 6 illustrates a case in which the masses are not in the same plane.

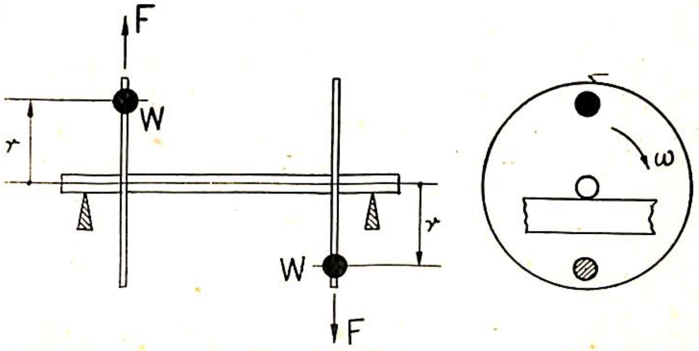

Fig. 6.

Two masses of weight W lbs. are each attached to perfectly-balanced discs on the same shaft, such that their radii of rotation are the same, r feet, and in end view these two masses are radially opposed.

If the shaft were mounted on two parallel knife-edged supports as shown, there would be no tendency for the shaft to roll, since the two masses produce equal and opposite turning moments in all angular positions of the shaft. The system is termed as being in "Static Balance."

However, if the shaft were rotating at a speed of w radians per second, two centrifugal forces, each equal to $\frac{W}{g}w^2r$ lbs., would act upon the shaft, parallel to each other and opposite in direction, thus forming a couple which would produce reactions at the bearings. This system, therefore, cannot be considered as being in balance when rotating. The term "Dynamic Balance" is reserved for the case in which a system of rotating masses does not produce reactions on its supports, as it can be seen that a system in static balance is not necessarily in dynamic balance.

When the masses all lie in the same plane of rotation, it is obvious that if they are in static balance they are also in dynamic balance, since no couples are produced by the centrifugal forces; but when the masses are not in the same plane of rotation, the couples produced have also to be dealt with, in addition to the centrifugal forces alone.

This necessitates the addition of two masses in different planes to such a system, so that the centrifugal forces, due to the balancing masses, will form a couple neutralising the original out-of-balance couple.

Before describing the method of balancing systems of rotating masses which are not coplanar, the method of determining bearing loads, due to known unbalance forces, will be outlined.

Loads on the Bearings of a Shaft carrying a Single Unbalanced Mass.

Referring to Fig. 7, a shaft, supported in bearings at A and B, carries an unbalanced mass of weight W lbs. rotating at a radius of r feet. The distances of the plane of rotation of this mass from A and B are a and b respectively.

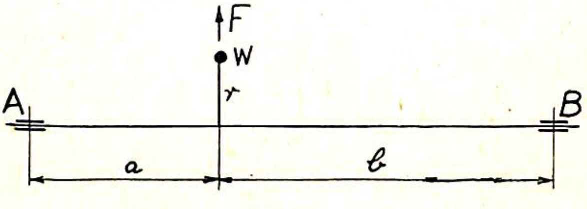

Fig. 7.

If the angular velocity of the shaft is w radians per second, the centrifugal force $F = \frac{W}{g}w^2r$ lbs. The shaft may be considered as a beam supported at points A and B with a single load applied at the centre of rotation of the mass W.

Hence the reaction at $\quad A = \dfrac{F\,b}{(a + b)}$

and the reaction at $\quad B = \dfrac{F\,a}{(a + b)}$

These reactions constitute the bearing loads due to the unbalanced mass W, and, of course, the direction of these loads rotates with the shaft, being always opposed to the centrifugal force F.

Loads on the Bearings of a Shaft carrying several Unbalanced Masses.

In Fig. 8, a rotating shaft supported in bearings at A and B carries masses at various points along its length, the relative angular positions of these masses being shown in the end view of the shaft.

Considering the effects of the individual masses on bearing B, the centrifugal forces are first calculated,

$$F_1 \text{ being } \frac{W_1}{g} w^2 r_1,\ F_2 \text{ being } \frac{W_2}{g} w^2 r_2, \text{ and so on.}$$

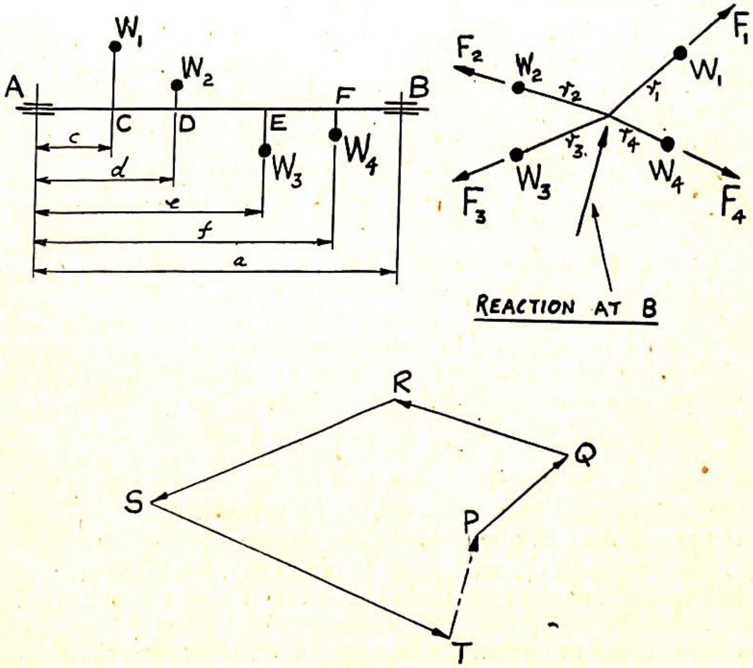

Fig. 8.

Then the reaction at B due to $F_1 = \dfrac{F_1 c}{a}$

$$\text{due to } F_2 = \frac{F_2 d}{a}$$

$$\text{due to } F_3 = \frac{F_3 e}{a}$$

$$\text{due to } F_4 = \frac{F_4 f}{a}$$

Since the denominator a is common to each, these reactions may be considered as proportional to the product of the centrifugal force and the distance of the corresponding plane of rotation from A,

i.e., reaction due to F_1 is proportional to $F_1 c$
reaction due to F_2 is proportional to $F_2 d$
etc.

Now, these reactions are all acting in different radial directions, in each case being opposed to the respective centrifugal force. Therefore, the resultant reaction at B must be obtained by making a vectorial summation as shown in Fig. 8.

Set out PQ in the direction of F_1, such that the length of PQ represents $F_1 c$ to a convenient scale. Then set out QR in the direction of F_2 such that the length of QR represents $F_2 d$ to the same scale. Similarly, proceed with RS and ST, representing $F_3 e$ and $F_4 f$ respectively.

Then TP is the vector representing the reaction at B, the actual value being obtained by dividing the quantity represented by TP by a.

This reaction will act in the direction TP relative to the centrifugal loads, as shown in the end view of the shaft.

It can be seen that this procedure is exactly the same as that which would be adopted to determine the reaction at a support of a beam carrying loads at various points along its length; *i.e.*, to determine the reaction at bearing B, moments are taken about A, the moment due to F_1 being $F_1 c$, due to F_2 being $F_2 d$, etc. The moment of the reaction at B must be equal and opposite to the sum of the moments of the loads, and the vector diagram of Fig. 8 is thus a "moment" vector diagram, TP representing the moment of the reaction at B, about A.

To determine the reaction at the bearing A, the same procedure is followed, moments this time being taken about B; then the moment of F_1 is $F_1\ (a-c)$, the moment of F_2 is $F_2\ (a-d)$, etc. These moments are again added vectorially, their directions again being the same as their corresponding centrifugal force, and the closing line of the vector diagram represents the moment of the reaction at A, about B.

Appreciating this method of determining the reactions at the bearings of shafts carrying unbalanced masses in different planes of rotation, we may now pass on to the problem of balancing such systems, since the problem resolves itself into the addition of rotating masses to the system, such that the reactions on the bearings are zero.

Balancing of a Single Mass by other Masses not in the same plane of rotation.

Referring to Fig. 9, AB is a shaft carrying an unbalanced mass of weight W_1, at a radius of rotation of r_1.

It is not always convenient to balance W_1 by a mass diametrically opposite, *i.e.*, in the same plane of rotation, and in such a case, the following procedure should be adopted.

Selecting planes of rotation at C and D as being suitable points for the addition of balancing masses, the shaft may be imagined as being supported in bearings at C and D, the actual positions of the bearings being ignored, since if there are no reactions on bearings at C and D, there will be no reactions at bearings anywhere along the shaft.

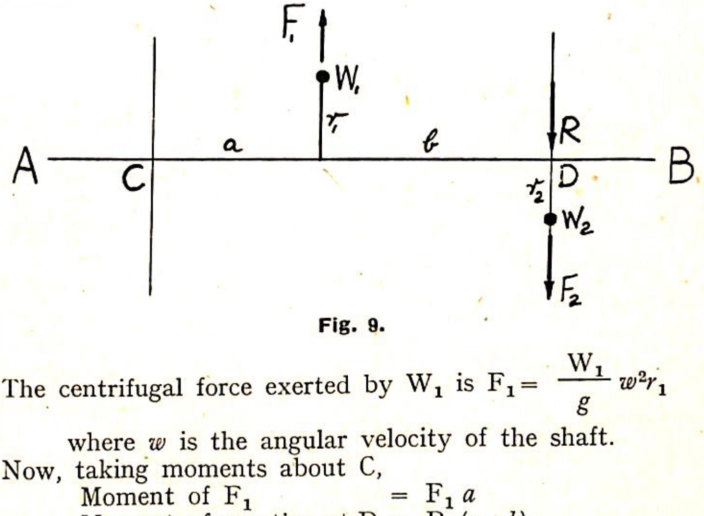

Fig. 9.

The centrifugal force exerted by W_1 is $F_1 = \dfrac{W_1}{g} w^2 r_1$

where w is the angular velocity of the shaft.

Now, taking moments about C,

Moment of F_1 $= F_1 a$

Moment of reaction at D $= R\,(a+b)$

R being the reaction at D.

These two moments are equal and opposite in direction,

therefore $F_1 a = R\,(a+b)$

$$\text{or} \quad R = \frac{F_1 a}{(a+b)}$$

To neutralise this reaction R, a mass of weight W_2 may be added to the shaft in plane D, such that its centrifugal force F_2 is equal to, and acting in the same direction as R, as shown.

$$\text{Then} \quad \frac{W_2}{g} w^2 r_2 = \frac{F_1 a}{(a+b)} = \frac{W_1}{g} w^2 r_1 \frac{a}{(a+b)}$$

$$\text{or} \quad W_2 r_2 = W_1 r_1 \frac{a}{(a+b)}$$

Taking moments about D, and adding a mass of weight W_3 in plane C, we get

$$W_3 r_3 = W_1 r_1 \frac{b}{(a+b)}$$

for no reaction at C.

It is not necessary to use a vector diagram in this case, since all the masses considered lie on the same line in end view, thus enabling the additions of the moments to be done arithmetically.

From the quantities $W_2 r_2$ and $W_3 r_3$ it can be decided what weights and corresponding radii of rotation are to be used in planes C and D to neutralise the reactions at C and D, thus balancing the system.

Balancing of a number of Masses rotating in different planes.

In Fig. 10, AB is a rotating shaft carrying unbalanced masses of weight W_1, W_2, W_3, etc., at radii of rotation r_1, r_2, r_3, etc., respectively. C and D are the points at which balancing masses are to be attached to the shaft, the distances between each point on the shaft at which masses are attached being shown in the diagram. The angular positions of the masses are shown in the end view of the shaft.

The first step is to find what force is required at D, such that when moments are taken about C, the vectorial sum of the moments shall be zero.

Now, Centrifugal force F_1 due to W_1 is proportional to $W_1 r_1$
,, ,, F_2 ,, W_2 ,, ,, ,, $W_2 r_2$

Therefore, taking moments about C,

Moment due to W_1 is proportional to $W_1 r_1 a$
,, ,, W_2 ,, ,, ,, $W_2 r_2 b$
,, ,, W_3 ,, ,, ,, $W_3 r_3 c$ etc.

Set out the moment vector diagram as follows :—

Draw PQ in the direction of the centrifugal force F_1 (see end view of shaft) such that the length of PQ represents $W_1 r_1 a$ to some convenient scale. Then draw QR in the direction of F_2 such that the length QR represents $W_2 r_2 b$ to the same scale. Continue

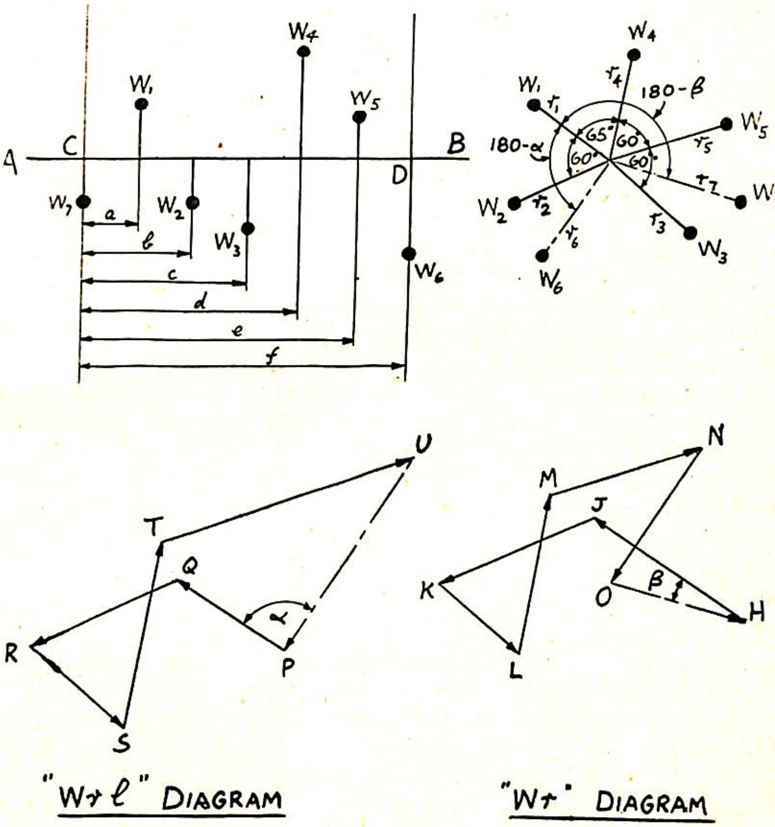

Fig. 10.

with RS, ST, and TU, representing $W_3 r_3 c$, $W_4 r_4 d$, $W_5 r_5 e$, respectively.

Then UP represents the quantity $W_6 r_6 f$ such that the vectorial sum of the moments is zero, where W_6 and r_6 are respectively the weight of the mass to be attached in plane D, and the radius at which it is to rotate.

The angular position at which W_6 is to be attached is given by the direction of the vector UP, *i.e.*, in the end view of the shaft, the direction of the centrifugal force due to W_6 is the same as the direction of UP in the vector diagram.

The second step is to find what force is required at C, such that when moments are taken about D, the vectorial sum of the moments shall be zero. The same procedure is followed in doing this vectorial summation, the various vectors being made to represent

$W_1r_1(f-a)$, $W_2r_2(f-b)$, etc. The closing vector in this case will represent W_7r_7f, where W_7 and r_7 are respectively the weight of the mass to be placed in plane C, and the radius at which it is to rotate.

Alternatively, this second step may be simplified as follows :—

Since the criterion for equilibrium of a system of forces is that two conditions shall be satisfied, namely, that the sum of the moments of the forces about any point, and also the sum of the forces themselves, shall both be zero, it is only necessary to consider the centrifugal forces alone, after the first step has been completed.

Therefore, to decide on the mass and radius of rotation for plane C, it is only necessary to draw a force vector diagram, not forgetting to include the force existing in plane D.

The second step may then be made as follows :—

Again referring to Fig. 10, set out HJ in the direction of F_1, such that the length of HJ represents W_1r_1 to some suitable scale. Then draw JK in the direction of F_2 such that the length of JK represents W_2r_2 to the same scale. Continue with KL, LM, MN, and NO, representing W_3r_3, W_4r_4, W_5r_5, and W_6r_6 respectively.

Then OH represents W_7r_7, where W_7 and r_7 are respectively the required weight and radius for plane C. The direction of OH gives the direction in which W_7 is to be attached.

Whichever method is used for the second step, both the quantity W_7r_7, and the direction of the closing vector giving this quantity, will be the same, and, if desired, both methods may be used, this serving as a check on the accuracy of the vector diagrams.

A worked example of the method of balancing is given as follows.

Referring again to Fig. 10, let the quantities involved be as follows :—

$W_1 = 10$ lbs. $r_1 = 2$ ft. $a = 1$ ft.
$W_2 = 20$ lbs. $r_2 = 6$ ins. $b = 3$ ft.
$W_3 = 15$ lbs. $r_3 = 1$ ft. 6 ins. $c = 5$ ft.
$W_4 = 5$ lbs. $r_4 = 3$ ft. $d = 6$ ft.
$W_5 = 10$ lbs. $r_5 = 2$ ft. 6 ins. $e = 8$ ft.
$f = 9$ ft.

The angular dispositions of all these masses are indicated in Fig. 10. It is convenient to arrange the necessary information in tabular form as shown.

The vector diagrams are shown in Fig. 11, UP representing a quantity 260, which, when divided by the distance f, 9 ft., gives $W_6r_6 = 28{\cdot}88$.

Now, if the radius r_6 is to be **2 feet**, **W_6 = 14·44 lbs.**

In the second vector diagram, NO represents W_6r_6, *i.e.*, 28·88, and OH represents W_7r_7, equal to 11·6.

If r_7 is to be **1 foot**, W_7 will be **11·6 lbs.**

Mass No.	Weight of Mass W lbs.	Radius of Rotation r feet.	Distance from C l feet.	W r	W r l
1	10	2	1	20	20
2	20	½	3	10	30
3	15	1½	5	22½	112½
4	5	3	6	15	90
5	10	2½	8	25	200

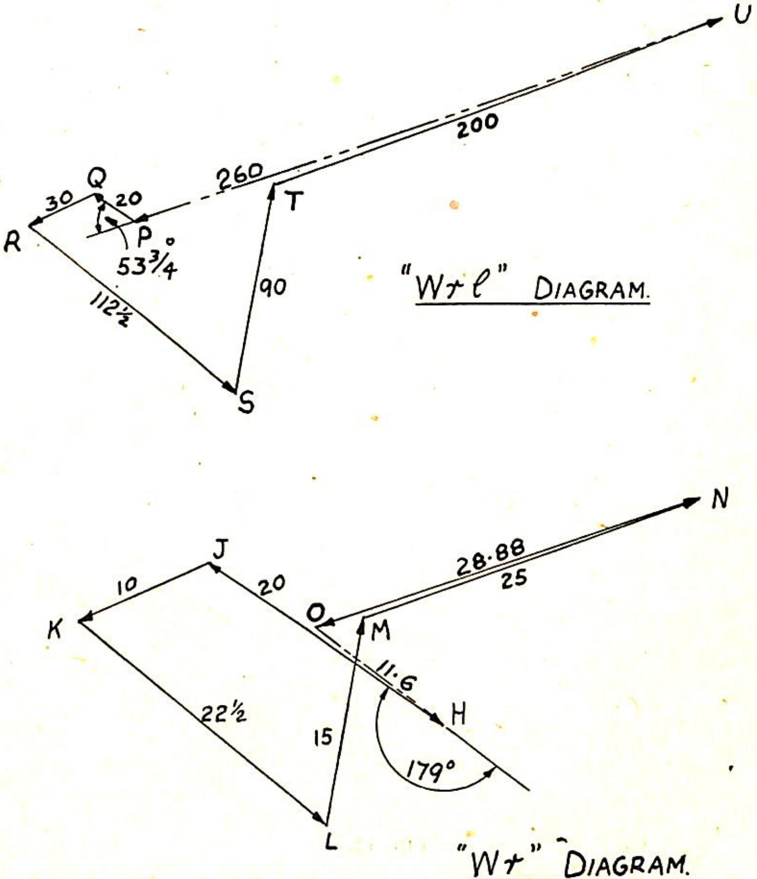

Fig. 11.

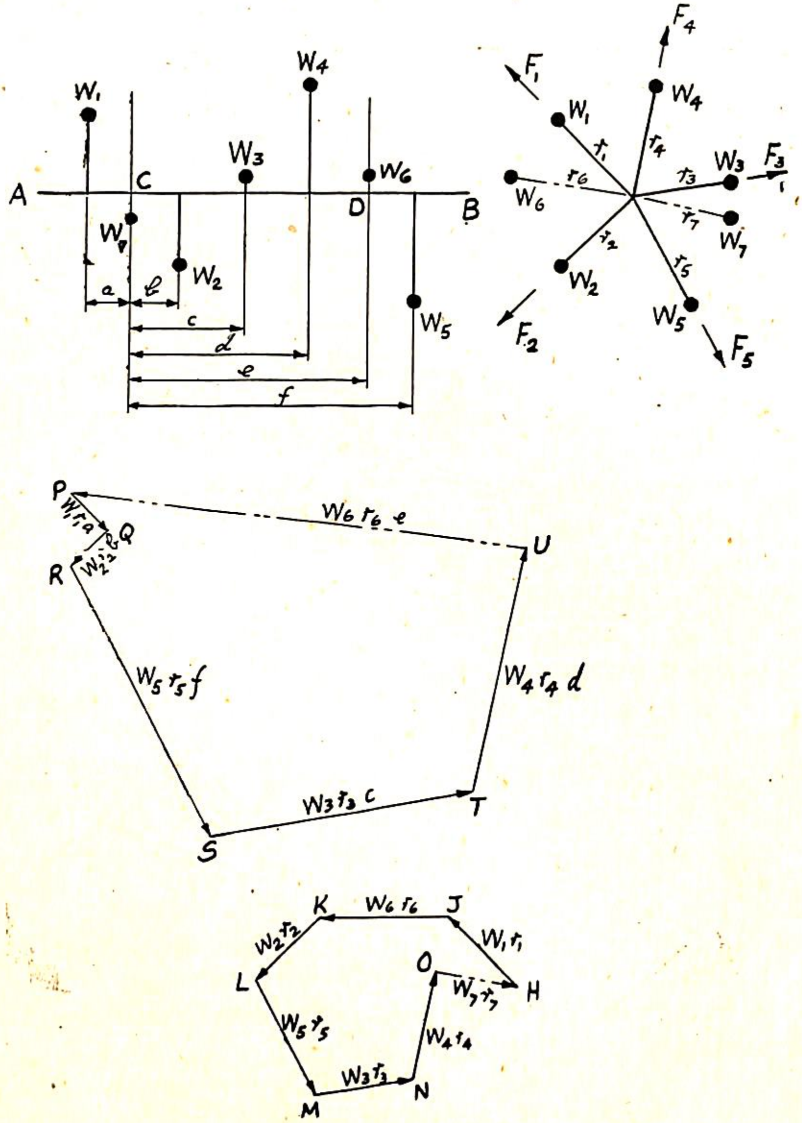
W₁
W₄
W₃
W₆
A
C
D
B
W₇
W₂
W₅
a
b
c
d
e
f
F₄
F₁
F₃
F₂
F₅
r₁
r₄
r₆
r₃
r₇
r₂
r₅
P
Q
R
S
T
U
W₁r₁a
W₂r₂b
W₆r₆e
W₅r₅f
W₄r₄d
W₃r₃c
K
J
L
O
H
M
N
W₆r₆
W₂r₂
W₁r₁
W₇r₇
W₅r₅
W₄r₄
W₃r₃

Fig. 12.

The angular disposition of the balancing masses is indicated on the vector diagrams in Fig. 11, the angle between W_6 and W_1 being $53\frac{3}{4}^\circ$, and the angle between W_7 and W_1 being 179°.

If it is not possible to choose the planes of rotation of the balancing masses such that they include all the unbalanced masses, as is illustrated in Fig. 12, W_1 and W_5 lying outside the balancing planes at C and D, then care must be taken to ensure that the correct directions of action are used for the construction of the moment vector diagram.

Referring to Fig. 12, if the shaft is imagined to be supported in bearings at C and D, then, taking moments about C, the reaction at D due to W_1 will be acting in the same radial direction as the centrifugal force F_1.

But the reaction at D due to W_2 will be acting in a direction radially opposite to the corresponding centrifugal force F_2, as will also apply to the reactions due to the masses W_3, W_4 and W_5.

Therefore, in laying out the moment vector diagram, the vector representing $W_1 r_1 a$ should be plotted in the *opposite* direction to the direction of the centrifugal force F_1, whilst the remaining vectors representing $W_2 r_2 b$, $W_3 r_3 c$, etc., should be plotted in the same direction as their corresponding centrifugal forces.

The force vector diagram used for the second step of the balancing operation is, of course, unaffected by the positions of the balancing planes C and D.

The vector diagrams for this case are shown in Fig. 12.

Best Positions for Balance Weights.

It is not always possible to choose positions for balance weights purely from the point of view of placing them in the most effective positions, since the design of the parts involved may dictate only a limited number of alternative positions. However, the weights should always be placed as far apart as possible, so as to gain the advantage that the large distance between them makes it necessary to use only relatively small balance weights to produce the required balancing couple. Therefore, when choosing the planes of rotation for the balancing masses, their positions should be as far apart as the design of the system will permit.

Relation between Centrifugal Force and Moment Vector Diagram.

Referring to Fig. 13, *a b c d* and A B C D are the force and moment vector diagrams respectively for the system of rotating masses shown, moments being taken about point X. The order of summation of the vectors has been chosen to avoid re-entrant angles in the diagrams. AB represents the quantity $W_1 r_1 l_1$ and *ab* represents $W_1 r_1$, therefore the ratio of the lengths AB/*ab* is equal to

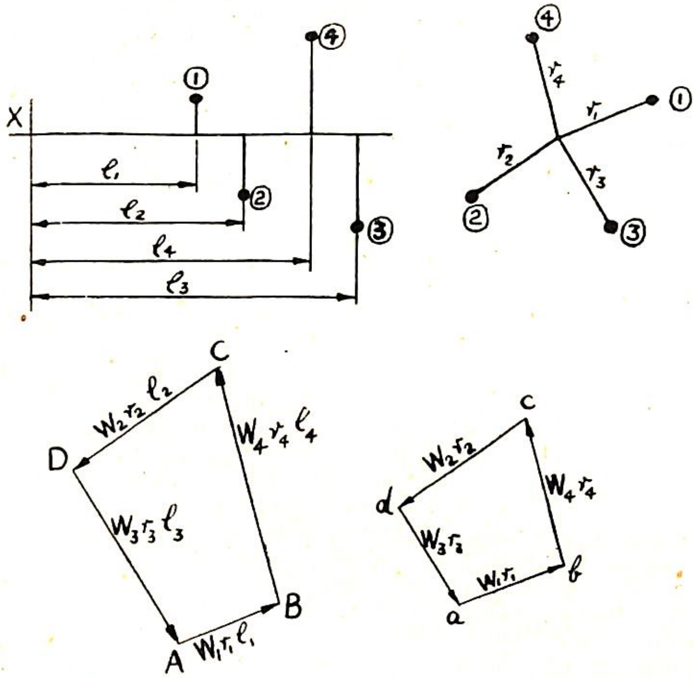

Fig. 13.

$$k \frac{W_1 r_1 l_1}{W_1 r_1} = k\, l_1,$$

where k is a constant, depending on the scales of the diagram.

Similarly, $BC/bc = k\, l_4$, $CD/cd = k\, l_2$, $DA/da = k\, l_3$.

If the polygons are similar, the above ratios will all be equal; *i.e.*, $l_1 = l_2 = l_3 = l_4$. This means that the masses all lie in the same plane of rotation.

In a system of two masses only, the foregoing principle clearly shows that balance is impossible unless the masses are in the same plane of rotation, since both the force and moment vector diagrams can only consist of two lines, one returning over the other, in which case the ratios of the sides would always be equal.

In a system of three masses, the force and moment diagrams will be triangles, and since no two triangles can be drawn dissimilar with their corresponding sides parallel, the ratios of the sides will be equal and hence it is possible for the three masses to be in balance only if they are in the same plane of rotation.

There is the case, however, of the diagrams consisting of three lines all colinear, as shown in Fig. 14, in which case the ratios of the lengths AB/*ab*, BC/*bc* and CA/*ca* can be different. This means that three masses not in the same plane of rotation can be in balance, provided that they all lie in a common plane which includes the axis of rotation. (This is the case of the single mass balanced by two other masses in different planes of rotation).

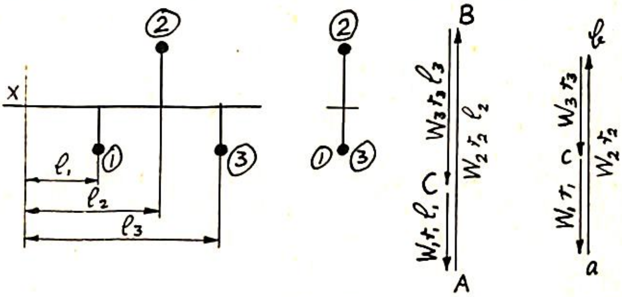

Fig. 14.

The force and moment diagrams for a system of four masses may be quadrilaterals, whose ratios of corresponding sides are not equal. This case is illustrated in Fig. 15, in which *a b c d* is the force diagram, and ABCD is the moment diagram for the system shown in Fig. 13, moments again being taken about point X. *ab* represents W_1r_1, *bc* represents W_4r_4, etc.

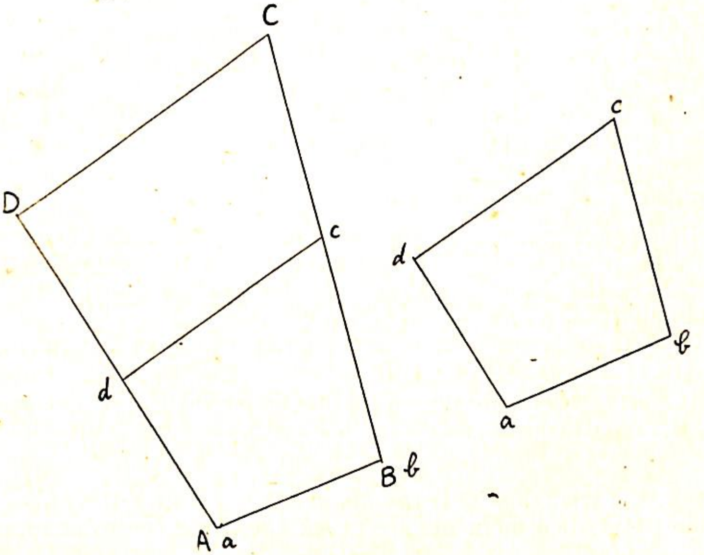

Fig. 15.

AB represents $W_1 r_1 l_1$, BC represents $W_4 r_4 l_4$, etc. Choosing the scales for these diagrams so that AB and *ab* are of equal length, then the two diagrams may be superimposed as shown.

Ratio $AB/ab = k\,l_1$, $BC/bc = k\,l_4$, etc., the value of the constant k again depending on the choice of scales.

It can be seen that for the above combination of fixed quantities, *i.e.*, four known unbalance forces at known distances from each other, there can be only one set of crank angles to give dynamic balance.

To illustrate this, the combined force and moment diagrams of Fig. 15 may be imagined as consisting of a framework of rigid members, pivoted at each junction. It would be impossible to distort this framework to give different relative angular positions to the members, therefore the angles of the diagrams are the only angles possible for dynamic balance.

This suggests a method for determining the axial locations of four known unbalanced masses with fixed relative angular positions, such that they shall be in dynamic balance. The four masses must, of course, be in static balance. Draw out the force vector diagram, as shown by *a b c d* in Fig. 15. Produce the sides *bc* and *ad* to C and D respectively, such that CD is parallel to *cd*, the position of CD being chosen at random. Then *ab*CD will be a moment vector diagram for the four unbalanced forces, provided that their axial positions conform to the conditions previously laid down, *i.e.*, the ratios of the sides AB/ab, BC/bc, CD/cd and DA/da should be used to determine the lengths l_1, l_4, l_2 and l_3 respectively.

Now, the ratio of the lengths $AB/ab = k\,l_1$, and since $AB/ab = 1$, then $k = 1/l_1$ so that $BC/bc = l_4/l_1$, $CD/cd = l_2/l_1$, and $DA/da = l_3/l_1$.

If the distance l_1 is taken as unity, then the ratios of the sides will give directly the position of the corresponding mass, *i.e.*,

$$BC/bc = l_4, \quad CD/cd = l_2, \quad \text{and } DA/da = l_3.$$

Significance of the position of the line CD.

Referring again to Fig. 15, the position for the line CD actually fixes the position of the point about which moments are being taken, *i.e.*, the distance l_1 in Fig. 13. Thus, by drawing the line CD in any position, a point is fixed about which moments shall be taken so that the moment vector diagram will have a form similar to ABCD.

If the line CD were drawn so that it passed through the point *a* of the force diagram, as shown in Fig. 16, than the ratio $aD/ad = 0$

$$i.e., \quad l_3 = 0,$$

so that the triangle *ab*C is the moment diagram for the forces (1), (2) and (4), when moments are taken about the centre of rotation of mass (3).

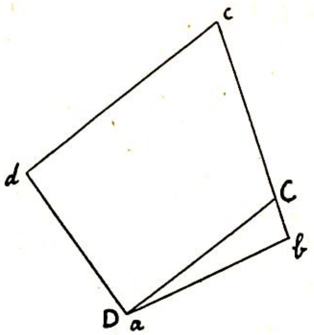

Fig. 16.

Geometric Construction for solving Four Crank Problems.

Another and better construction for applying to problems involving four rotating masses is shown in Fig. 17.

To determine the axial locations of four known unbalanced masses, such that they shall be in dynamic balance, the following procedure is adopted.

EFGH is the force vector diagram for the system shown, EF representing W_1r_1, FG representing W_3r_3, GH representing W_2r_2 and HE representing W_4r_4. Taking the point of intersection E of the two shorter sides, draw the diagonal HF opposite to this point, and draw lines from E parallel to HG and FG, cutting HG and FG in h and f respectively. The intersections of Eh and Ef with HF are x and y respectively.

Then the line HF represents the distance between masses (1) and (4) to some scale, x and y, being the points at which masses (2) and (3) respectively are to be attached, so that the system will be in dynamic balance.

The proof of this construction is as follows :—If moments are taken about the centre of rotation of mass (1), the triangle HEh is a moment vector diagram for the other three forces, since the line Eh is parallel to FG, and as shown in the previous article, the diagram HEh is the moment vector diagram for moments taken about the centre of rotation of mass (1), as the parallel to FG passes through the point E. Similarly, if moments are taken about the centre of rotation of mass (4), the triangle EFf is the moment vector diagram for forces (1), (2) and (3), Ef being parallel to HG.

$$\text{Then } Hh/HG = k\,b$$
$$\text{and } Ff/FG = k\,(d-c)$$

The constant k in this case is determined as follows :—In the moment vector diagram HEh, HE represents W_4r_4d. In the force

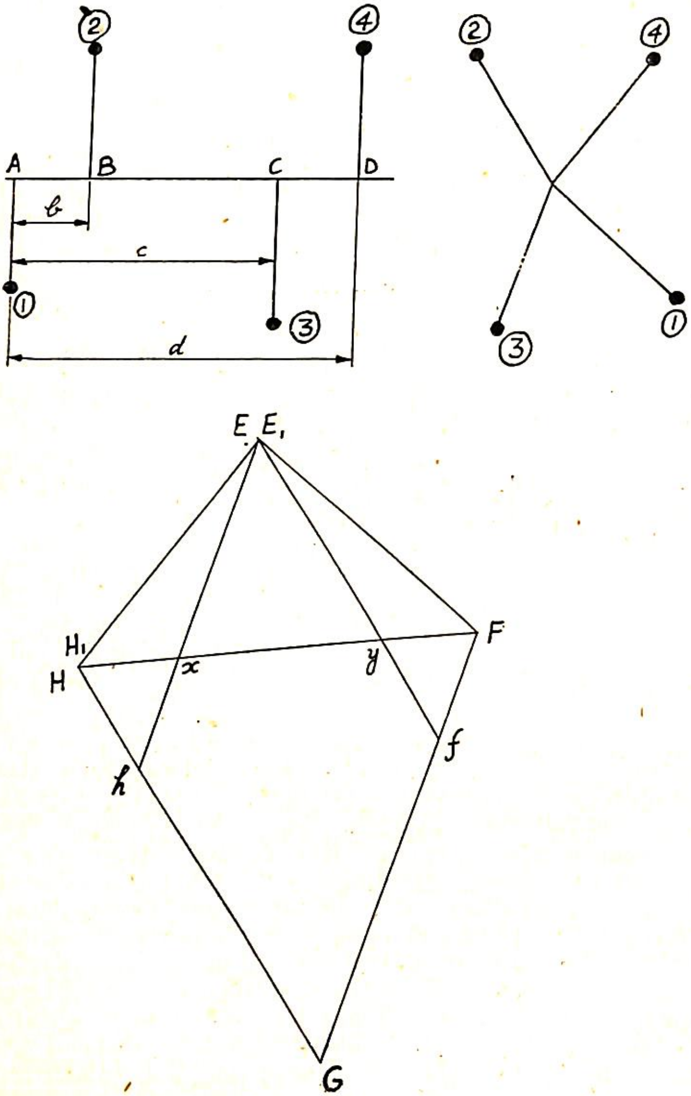

Fig. 17.

vector diagram HEFG, HE represents W_4r_4. The ratio of the lengths of these sides is unity, therefore

$$1 = k\,d \quad \text{or} \quad k = 1/d$$

$$\text{Then } Hh/HG = b/d \quad \text{and} \quad Ff/FG = \frac{(d - c)}{d}$$

Now, in the triangle HFG, hx is parallel to FG.

$$\therefore \quad Hx/HF = Hh/HG = b/d$$

Also yf is parallel to HG.

$$\therefore \quad \frac{Fy}{FH} = \frac{Ff}{FG} = \frac{(d - c)}{d}$$

It can be seen, therefore, that the points H, x, y, and F give the relative axial locations of the four masses such that they shall be in dynamic balance.

It is possible to use the same construction to determine the crank angles required to place a system of four masses, with fixed relative axial positions, in dynamic balance. In this case, referring again to Fig. 17, the positions of the centres of rotation of the masses are given, being H, x, y and F.

Take any point E at random, and draw the lines EH, Ex, Ey and EF. Then draw HG parallel to Ey and FG parallel to Ex.

Then the quadrilateral EFGH is the force vector diagram for four masses in the planes at H, x, y and F. The directions of the sides of this quadrilateral give the crank angles, and the lengths of the sides give the relative Wr values, for the four masses such that they shall be in equilibrium.

It is important to note that in both applications of the above construction, there is a definite relationship between the positions of corresponding force vectors and centres of rotation ; *i.e.*, the force vector HE in Fig. 17 is the vector for the outermost mass on the *opposite* side, mass number (4). Similarly, EF is the vector for force number (1). Also GH is the vector for the inner mass on the *same* side, mass number (2) ; and FG is the vector for mass number (3).

Unavoidable Unbalance.

The foregoing principles may be applied to any system of rotating masses to calculate the magnitude and position of the balancing masses which have to be added to the system to reduce the resultant unbalance forces to zero.

However, in practice it is often found that even with the most careful machining and assembly, a system which has been designed correctly may still suffer from a certain amount of unbalance. At high rotational speeds, this small remaining unbalance may cause objectionable vibrations which should be eliminated. This elimination can only be carried out experimentally, using dynamic balancing machines, a few types of which will be described later.

This imperfection of balance is due in most cases to slight dimensional errors arising during machining or assembly operations, or it may be caused by lack of homogeneity in the materials used for the various rotating parts. For example, in the case of the rotor of an alternator, the machining may be carried out perfectly, but unbalance forces may arise due to the windings laid in the rotor slots not being in perfect balance themselves. It can be realised that to assemble these windings so as to be in perfect balance, is as nearly impossible as to make the attempt unprofitable, and consequently the finished rotor complete with windings is tested for balance in a dynamic balancing machine, and any unbalance found to exist is eliminated by adding counterbalancing weights to the rotor, or alternatively, material may be removed by drilling holes (or by some other means) at the required positions.

It is impossible to dynamically balance a rotating system which contains an unseen unbalance by any means other than running it at speed in a dynamic balancing machine. This may be appreciated if one considers the case of a cylindrical part rotating about its longitudinal axis. The cylinder may be considered as a number of discs all mounted on the same shaft. Each disc will have its own centre of gravity, and except in the case of perfect machining and perfectly homogeneous material, these centres of gravity will all lie in different radial directions and at varying distances from the axis of rotation. Consequently, each disc will have its own unbalanced centrifugal force, these forces being undetectable individually, the only measurable quantity being their *combined* effect on the rotor. It has already been shown that it would be possible for a rotor to be in static balance and yet not in dynamic balance, therefore the rotor has to be run at speed in a dynamic balancing machine, which is a machine which is capable of detecting the combined effect of the unbalance forces in the whole rotor.

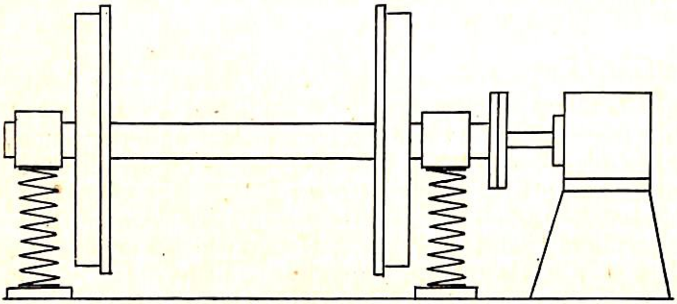

Fig. 18.

Dynamic Balancing Machines.

Various forms of dynamic balancing machines have been developed, and the following descriptions cover only a few of the many types which are at present in existence.

As has already been stated, the function of a dynamic balancing machine is to detect the out-of-balance forces existing in a rotating part. One of the simplest forms of dynamic balancing machine is diagrammatically illustrated in Fig. 18, and a machine of this type was used by the L.M.S. Railway Co. to balance railway carriage wheels.

The part to be balanced, in this case a wheel and axle assembly, is supported in bearings mounted on springs as shown. The axle is rotated by an electric motor or pulley shaft through a flexible coupling. This flexible coupling ensures that any extraneous vibrations are not transmitted to the part being tested. Any unbalance existing imposes vibrating loads on the spring-supported bearings, which then oscillate. By running the axle up to a speed above the natural frequency of oscillation of the springs, and then allowing the speed to fall slowly, the spring oscillations will be quite pronounced when the speed drops to resonant speed. The size and position of the required balance weights is found by trial and error, balancing plates being attached at different points on the faces of the wheels near the rims. This trial and error is a feature of these simple types of machine, but other types incorporate systems whereby the trial and error is eliminated, the unbalance forces being measured and located directly.

Dynamic Balancing Machine for Small Parts.

The machine described here was produced by Messrs. Carl Schenk, Ltd., of Darmstadt, Germany, and is suitable for use on small mass-produced parts such as the armatures of small electric motors. It was made in two sizes, one being suitable for rotors up to 2¼ lbs. weight and the other for rotors up to 11 lbs. weight. The essentials of the machine are shown diagrammatically in Fig. 19.

Two tubular side members A and B are connected transversely by pieces C and D, which each carry two ball-bearing rollers. These rollers carry the shaft of the rotor being tested. The transverse member C is supported by thin steel spring plates which are carried on the saddle E, and this saddle may be moved longitudinally to suit rotors of different lengths. The other end of the frame is supported on vertical helical springs F, which may be adjusted by means of the knurled knob G.

The arrangement is such that the frame can oscillate at the sprung end, the thin steel plates acting as hinges.

A lever H at the spring end of the frame registers with another lever which carries a mirror K, which reflects light from the lamp L

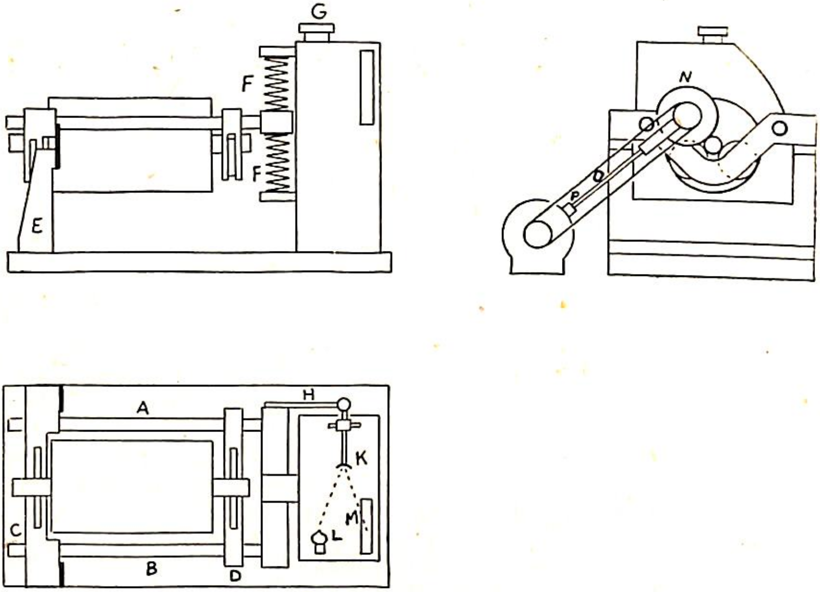

Fig. 19.

on to a translucent scale M, thus enabling the amplitude of the oscillations to be observed.

The rotor to be balanced is placed with its shaft resting in the rollers, and is rotated by means of a friction wheel N, which is brought into contact with the periphery of the shaft by means of the swinging arm O. The friction wheel is driven by an electric motor through a belt, and either direction of rotation may be applied to the rotor by turning the friction wheel in its pivot P, so that the belt is either crossed or open.

Before commencing a test, the rotor is placed on the rollers, and the scale reading on M is brought to zero by adjusting the knob G. The test then proceeds. The rotor is run up to a speed of about 1000 R.P.M., the friction wheel is then raised and the rotor is allowed to slow down. As it passes through the speed which coincides with the frequency of vibration of the frame, the oscillations become pronounced and the amplitude of these oscillations is read off the scale. The direction of the unbalance force is determined by bringing a stationary scriber into contact with the shaft during its resonant period, from a vertical direction, *i.e.*, in the plane of oscillation. The shaft is previously painted by a quick-drying paint, usually chalk and spirit, in the region at which the scriber is to be applied, and due to the oscillation of the shaft, only a small arc is marked on the periphery of the shaft. The position of this arc indicates the direction in which the out-of-balance force lies, radially. The test is then repeated with the direction of rotation reversed, and although theoretically the results should be identical with either direction of rotation, the mean of the two sets of readings is taken for estimating the size and position of the required balance weight.

The rotor is then reversed end to end in the rollers, and the test is repeated, observations being taken as before. It should be ensured that the planes in which balance weights are to be added coincide with the plane through the axis of oscillation of the frame, *i.e.*, the plane through the spring plates. This is done by means of stops (not shown in the diagram) brought to bear against the ends of the shaft, with the shaft in its correct longitudinal position. The significance of this precaution will be shown later.

The method of dealing with a large batch of similar rotors would be to set the machine for running with the rotor in one position, and then proceeding to deal with each of the rotors in turn in that position until the whole batch was completed. The machine would then be set for running with the rotors reversed end to end, and the rotors would again be dealt with one by one in this second position. Each rotor would then bear scribed marks at each end of its shaft, showing the direction of unbalance, and the scale reading M could also be recorded on the rotor itself at the correct end.

The scale M may be calibrated by adding known weights to a rotor which is in itself perfectly balanced, at the radius at which balance weights are to be added on production pieces. This would enable the scale readings to be observed for known unbalance forces, and a curve could be drawn between scale reading and unbalance weight for each type of rotor to be tested. The operator could then convert the scale reading into weight units directly.

By ensuring that the planes in which balance weights are to be added coincide with the plane through the spring plates, we are merely following out the same procedure as was adopted in the case of finding required balance weights for a known system of rotating masses, *i.e.*, we actually take moments about these planes of rotation in turn, and the scale reading M gives the weights required in these planes.

60-Ton Dynamic Balancing Machine.

This machine was constructed by Messrs. Carl Schenk, Ltd., for the Victoria Falls and Transvaal Power Company, Ltd., to enable them to balance alternator and turbine rotors at their shops at Rosherville. At the time it was constructed it was the largest dynamic balancing machine in existence, and the author has no knowledge of any larger one having been built since. It is capable of balancing rotors from 3½ tons up to 60 tons in weight, with a maximum diameter of 11 feet 2 inches, and a total length between bearing centres not exceeding 20 feet.

The essential features of the machine are indicated in Fig. 20.

The massive bedplate A is rigidly supported on concrete foundations, since it is important that no vibrations should exist due to extraneous causes. Mounted on one end of the bedplate is a 500-H.P. electric motor M and the gear-box P through which the rotor being tested is driven. An electro-magnetic clutch T is used to couple the gear-box to the rotor. The rotor is supported on two headstocks B, which may be moved longitudinally to suit rotors of different lengths. These headstocks each support spherically seated bearings C, by means of a thin vertical steel plate D, which allows the rotor to oscillate transversely across the machine, the amplitude of these oscillations being limited by springs and stops. The spherically-seated bearings allow either end of the rotor to oscillate independently, the angular displacement of the rotor being provided for by a slight rotation of the bearing bushes in their spherical seats.

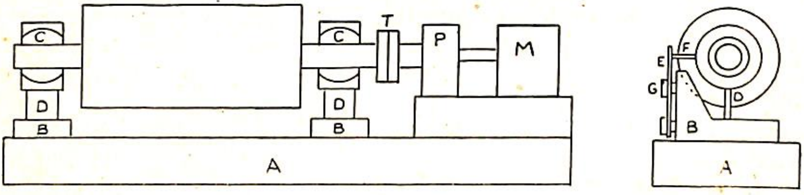

Fig. 20.

To carry out a balancing test, one bearing is firmly clamped in its central position, its only freedom of movement being angular displacement of the bush in its spherical seat. At the other end, the bearing is free to oscillate transversely, and a coloured pencil is held against the rotor shaft near the free bearing whilst the rotor is running at speed. The pencil marks the rotor shaft over only a short arc, and this indicates the direction in which the out-of-balance force lies. The amplitude of the oscillation is also recorded, and this provides the means for estimating the weight required to balance the rotor.

If the rotor were only slightly out of balance, the oscillations would not be sufficiently pronounced under ordinary conditions, therefore the rotor is run at a speed which coincides with the natural frequency of the support. This is accomplished by first locking both bearings in their central position and running the rotor up to a speed in excess of the frequency of the bearing support.

One bearing is then released, and the rotor is disengaged from the gear-box by means of the electro-magnetic clutch, and as the rotor slows down, it passes through the resonant speed, thus enabling measurements of the oscillations to be made. In practice, a second run is made with the direction of rotation reversed, the two sets of observations giving a better indication of the direction of the out-of-balance force.

A temporary balance weight is then added to the rotor to eliminate these oscillations, and the bearing originally free is then clamped, and the other one unclamped ; then the test is repeated, the oscillations of the other end of the rotor being observed. The weight necessary to neutralise oscillations at this end is then added, and as this weight will upset the balance at the other end, a third test is carried out with the originally free bearing again free, and the other one clamped. The oscillations this time will be found to be much less than at first, and a smaller weight is added to eliminate these vibrations. Again, this will theoretically upset the balance at the other end, but the effect is generally found to be negligible.

This approximation is necessary because balancing weights cannot be added in the planes through the axis of oscillation, *i.e.*, planes through the bearing centres.

In the end view of the machine is shown the thin plate supporting the bearings, and in addition another spring plate E is shown, which, through the thin flexible plate F, prevents the bearings from falling sideways. The plate E may be replaced by others of different stiffness, thus giving different ranges of resonant speed. Further adjustments for this periodicity of vibration may be carried out by altering the position of a clamp G, which secures the plate to the headstock.

As already stated, the driving motor is rated at 500 H.P., this large power being necessary owing to the fact that with alternator rotors there is a possibility that the windings may move when the rotor is running at speed, and, therefore, after balancing such a rotor, it is run up to 20% overspeed, and then again tested for balance.

Direct Measurement Dynamic Balancing Machine.

The principle of this machine is illustrated diagrammatically in Fig. 21, the machine being manufactured by Messrs. W. & T. Avery, Ltd., and is known as the Avery Dynamic Balancing Machine. The main feature of the machine is that it can give an immediate quantitative value and angular location to the required balancing compensation, this being achieved by comparative means. It is suitable for dealing with parts from $3\frac{1}{2}$ ozs. to 10 lbs. in weight, but similar machines of larger capacity are made by Messrs. Averys.

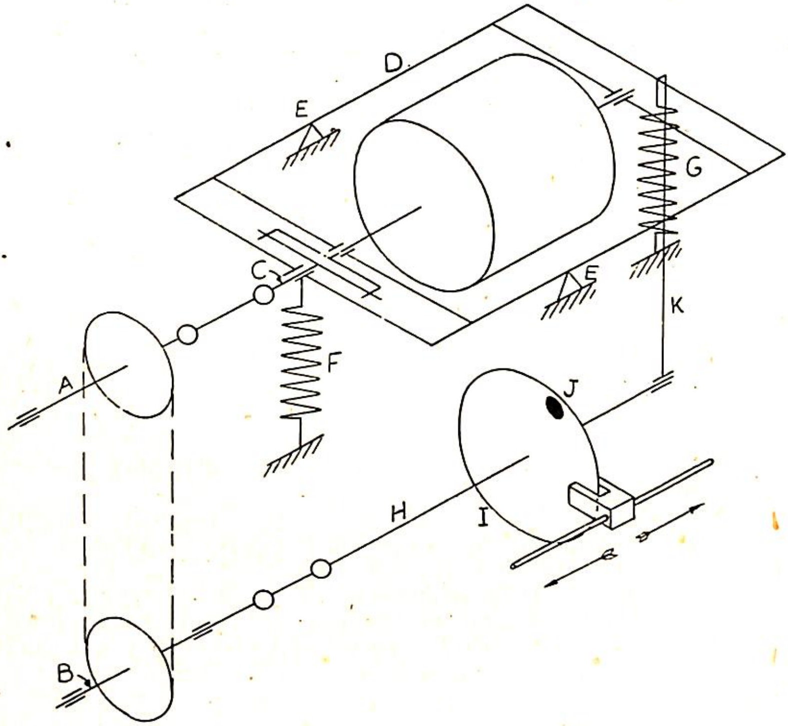

Fig. 21.

The shaft A is motor driven, and is coupled to the rotor shaft through universal flexible joints and a pin-type coupling, which ensures that no extraneous vibrations are imparted to the rotor. The countershaft B is actually geared to A to ensure synchronous running. The shaft C is carried in a bearing at one end of a cradle D, which is mounted on knife edges E. Cross members support the rotor shaft in bearings, and the longitudinal positions of these members may be varied to suit rotors of different lengths.

The side and cross members are rigidly connected to form the cradle, which is supported by springs F and G. Thus the cradle and rotor can oscillate about the knife edges, under the influence of out-of-balance forces. The knife edges may be traversed longitudinally to vary the fulcrum point of oscillation. Both of the springs F and G are adjustable for stiffness, this making it possible for the natural frequency of oscillation of the cradle to be varied.

The disc I rotates synchronously with the rotor, and carries a known out-of-balance weight J. A rod K couples the free end of shaft H to the end of the cradle, and this rod is supported in a vertical guide so that the cradle D and shaft H may oscillate together. The angular position of the weight J may be varied independently of the rotor, and in addition, the disc I may be traversed longitudinally.

It can be seen that by varying the longitudinal and angular locations of the disc I, an out-of-balance force may be produced which directly opposes the out-of-balance force of the rotor. The state of equalisation of these two forces is indicated when all vibration of the cradle ceases. An ingenious system (not shown in the diagram) converts all the adjustments necessary to eliminate vibration into scale readings, giving the required balancing weight directly, the angular location of these weights being shown by the position of the weight J relative to the rotor.

The knife edges E are adjusted so that they lie in one of the two planes in which compensating weights are to be added ; other settings of the machine are then made in accordance with the position of the other balancing plane. After determining the compensating weight to be added in this plane, it is necessary to either reverse the rotor end to end or move the knife edges E so that they lie in the other plane, and also reset the machine to suit the position of this plane.

As with the machines previously described, the rotor is run at the speed in resonance with the cradle oscillations. The oscillations are shown on special oscillation indicators incorporated in the machine.

Electrical Type Dynamic Balancing Machine.

The machine described here is known as the "Gisholt Type S" Dynamic Balancing Machine (Messrs. Burton Griffiths & Co. Ltd.). By correctly adjusting the machine, the amount of unbalance may be read directly off a meter, whilst its position is indicated stroboscopically.

Referring to Fig. 22, which is a diagrammatic representation of the machine, the part to be balanced is driven by belt drive from an electric motor (not shown) and runs in flexibly-supported bearings A and B. Attached to these bearings are coils C and D, which move in the fields of permanent magnets under the influence of any vibrations of the bearings. The voltages induced in these coils are proportional to the amplitude of the oscillation of the corresponding bearing.

The planes in which balance weights are to be added are indicated by XX and YY. Now, if the only unbalance inherent in the rotor existed in plane YY, then rotation of the rotor would produce a vibration of B, generating voltage b, and a lesser vibration of A, generating voltage a. By means of the voltage divider E, a

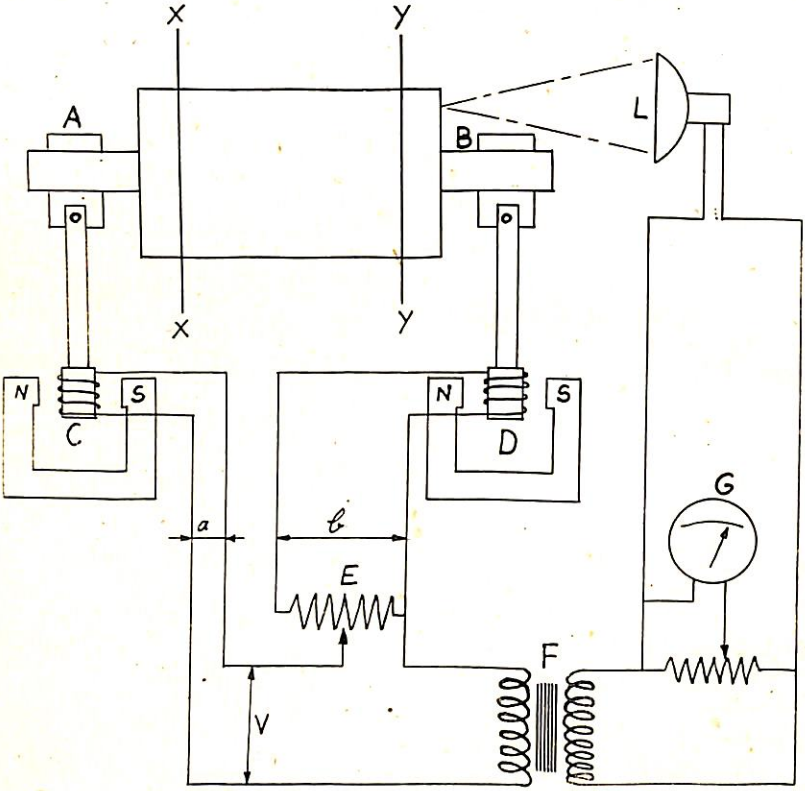

Fig. 22.

portion of *b* may be selected so as to be equal to *a* but opposite in direction, so that the resultant voltage V will be zero due to unbalances of any magnitude in plane YY.

With the voltage divider set in this position, any unbalance existing in plane XX will produce a larger voltage *a* and a correspondingly smaller voltage *b*, and the resultant voltage V will then be made up of the original *a* and part of the original *b*, which in themselves neutralise each other, and the voltages *a* and part of *b* due to the unbalance in plane XX. Thus, by selecting the correct position for the voltage divider, the effects of unbalance in plane YY are eliminated. The resultant voltage V is amplified by F, and the output is recorded on a meter G, which may be calibrated so as to give the necessary balancing weight directly.

To determine the balance weight required for plane YY, another position must be selected for the voltage divider, such that any unbalance in plane XX does not affect the meter reading, which will then give the balance weight for plane YY.

The angular location of the unbalance in each case is determined stroboscopically by means of the lamp L. A band of paper bearing numerals may be attached to the rotor, so that the portion of the rotor which appears to stand still in front of the lamp may be easily identified. This indicates the direction in which the balancing correction has to be made.

Composite Systems.

It is sometimes necessary to make a rotating component in two or more pieces, an example of this being the impeller and guide vanes of an aero engine supercharger rotor. If the pieces are not to be interchangeable with other units, it may only be necessary to dynamically balance the complete assembly, but if this course is adopted, some precaution should be made to ensure that after balancing the pieces will always be assembled in the same relative positions as they occupied whilst being balanced. The reason for this precaution is illustrated in Fig. 23.

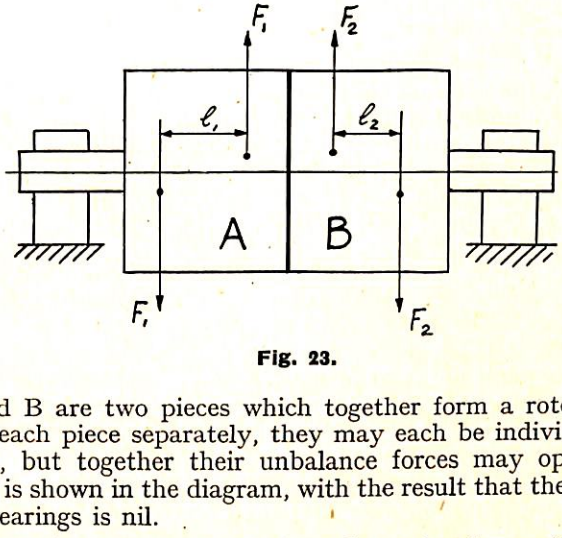

Fig. 23.

A and B are two pieces which together form a rotor. Considering each piece separately, they may each be individually unbalanced, but together their unbalance forces may oppose each other, as is shown in the diagram, with the result that the net effect on the bearings is nil.

In piece A there is an unbalanced couple of magnitude $F_1 l_1$, and in piece B there is an unbalanced couple $F_2 l_2$. If $F_1 l_1$ is equal in magnitude to $F_2 l_2$ but acts in direct opposition to $F_2 l_2$, then the whole rotor will be in equilibrium. But if piece B were assembled in a different angular position relative to A, so that the two couples were no longer acting in the same plane, there would be a resultant out-of-balance couple produced. Therefore, it should be ensured that the pieces A and B always occupy the same relative positions as when being balanced.

It can be seen that although the whole rotor is in balance, the individual out-of-balance couples impose a bending moment on the shaft, and if it is decided that this bending moment may be objectionable, then the pieces should be dynamically balanced separately.

It is hardly necessary to add that for interchangeable units, dynamic balancing of the individual pieces is essential.

Design Notes.

As has already been stated, it is advisable to select the planes in which balance weights are to be added to a rotating part, as far apart as possible. Therefore in the design of a part which will have to be dynamically balanced, provision should be made as near each end of the part as possible for addition of balance weights. Alternatively to adding balance weights, the required compensation may be effected by removing material by drilling holes, or by other means, at a point diametrically opposite the point at which weights would be added. This possibility should be taken into account in the design of the part.

Although no mention has been made of the effects of flexibility of shafts on the running characteristics of rotating parts, it should be borne in mind that further design work may be necessary in some cases to cover the possibility of torsional oscillations, which may be present even with correctly-balanced parts. The subject of torsional oscillations, however, is outside the scope of this pamphlet, but it can be stated here that consideration of this possibility may influence the choosing of the positions of the planes in which balance weights are to be added.

Conclusion.

The foregoing notes have never before, to the author's knowledge, been presented in the form of a complete paper on dynamic balancing. The theoretical principles involved are, in most text books, merely a preliminary to the more comprehensive problem of engine balancing, whilst the information on dynamic balancing machines is usually only available as isolated articles dealing with the machines themselves.

It is the author's hope that the usefulness of the contents of this paper to designers and draughtsmen will be comparable to the pleasure taken by him in the preparation of the notes.

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26. 1″ ,, ,, ,, (30 ,,).
27. ¾″ ,, ,, ,, (40 ,,).
28. ⅞″ ,, ,, ,, (40 ,,).
29. 1″ ,, ,, ,, (40 ,,).
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